Long-Range Dependence and Multiple Change-Points in Multivariate Time Series

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Purpose of this work

- Extend previous works on multiple change–points detection for univariate time series (Lavielle 1999, Lavielle and Teyssière 2005)

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- Consider a multivariate framework (direct extension since using a Gaussian contrast estimator)
- Present an adaptive method for finding the number of change-points, (i.e., the dimension of the model)

Remark

This work is the other side of the coin of the previous paper by Surgailis *et al.* (2008).

Standard univariate approach

In the univariate case : standard procedures for detecting single changes in variance:

- Change in variance for iid observations: Inclan and Tiao (1994)
- Change in variance for weakly dependent observations, ARCH(∞): Kokoszka and Leipus (1999)

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- Assumption of single change-point not realistic for financial time series
- How to deal with the case of multiple change points?

The multiple change-point case: the local approach

Binary segmentation algorithm (Vostrikova, 1981)

1. Apply the testing procedure on the whole sample

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Remark

This is a local detection algorithm

Empirical example: the FTSE 100 index

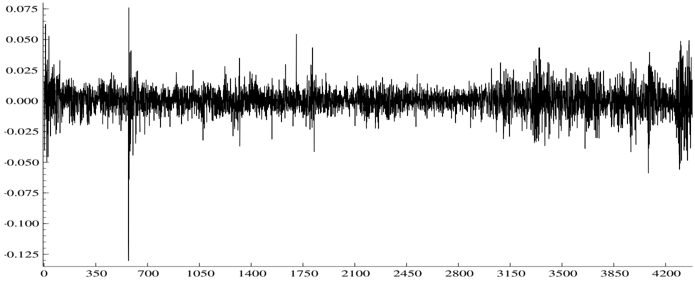
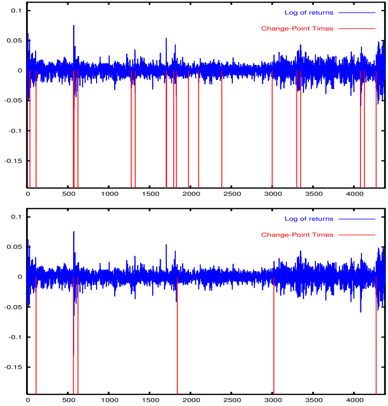


Figure: Log of returns on the FTSE 100 index $r_t = \log(P_t/P_{t-1})$ (1986–2002)

Remark

This series displays intermittency

Is the binary segmentation procedure reasonable?



- Top: Binary segmentation procedure
- Bottom: Global (adaptive) method presented later

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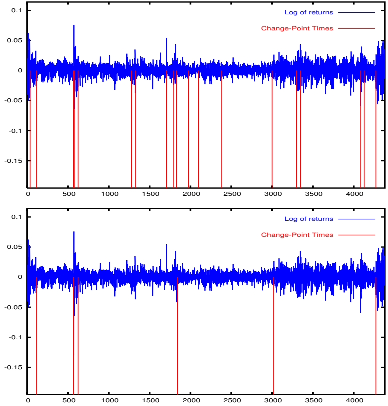

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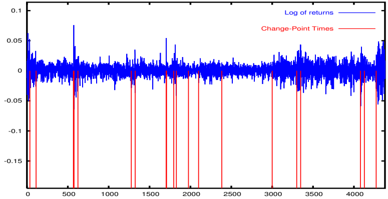

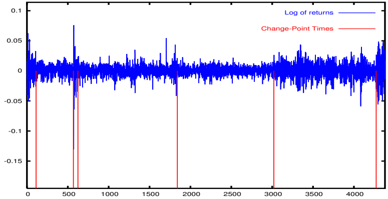

- Top: Binary segmentation procedure
- Bottom: Global (adaptive) method presented later
- There is a difference in the resolution
- Which dimension is the right one ?

Finding the dimension of the model

- Trade off between high resolution and parsimonious representation of the process

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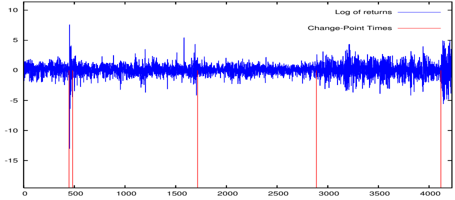
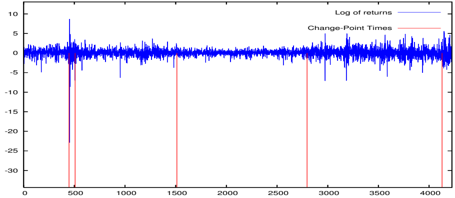
Finding the dimension of the model

- Trade off between high resolution and parsimonious representation of the process
- We wish to capture the "main" features of the model
- Method used: penalized likelihood function
- How to choose the penalty parameter ?

Multivariate change–points detection: Motivation

Empirical examples: the FTSE 100 and S&P 500 indices (1986–2002)





Adaptive detection of multiple change-points in variance (univariate case)

- Top : log returns on FTSE 100
- Bottom : log returns on S&P 500

Remark

Change-point times in the two series look very similar

Global detection method

- m–dimensional process $\{\mathbf{Y}_t = (Y_{1,t}, \ldots, Y_{m,t})'\}$ changing abruptly
- Process characterized by a parameter $\theta \in \Theta$ constant between two changes
- Let K be an integer and $\boldsymbol{\tau} = \{\tau_1, \tau_2, \ldots, \tau_{K-1}\}$ be an ordered sequence of integers verifying $0 < \tau_1 < \tau_2 < \ldots < \tau_{K-1} < T$.
- For all $1 \leqslant k \leqslant K$, define a contrast function $U(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k}; \theta)$ for estimating the parameter on the k^{th} segment
- Minimum contrast estimator of $\hat{\theta}(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k})$ on the k^{th} segment of $\boldsymbol{\tau}$, is defined as the solution to the minimization problem:

$$U\left(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k}; \hat{\theta}(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k})\right) \leqslant U(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k}; \theta) \quad \forall \theta \in \Theta.$$

Global method: Contrast function I

- For all $1 \leqslant k \leqslant K$, define G as follows:

$$G(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k}) = U\left(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k}; \hat{\theta}(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k})\right).$$

- Define the contrast function $J(\boldsymbol{\tau}, \mathbf{Y})$:

$$J(\boldsymbol{\tau}, \mathbf{Y}) = \frac{1}{T} \sum_{k=1}^{K} G(\mathbf{Y}_{\tau_{k-1}+1}, \ldots, \mathbf{Y}_{\tau_k}),$$

with $\tau_0 = 0$ et $\tau_K = T$.

Global method: Contrast function II

- We consider changes in the covariance matrix of $\{\mathbf{Y}_t\}$
- Assume that there exists
  - an integer $K^\star$,
  - a sequence $\boldsymbol{\tau}^\star = \{\tau_1^\star, \tau_2^\star, \ldots, \tau_{K^\star}^\star\}$ with $\tau_0^\star = 0 < \tau_1^\star < \ldots < \tau_{K^\star-1}^\star < \tau_{K^\star}^\star = T$
  - $K^\star$ $(m \times m)$ covariance matrices $\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, \ldots, \boldsymbol{\Sigma}_{K^\star}$ such that $\mathrm{Cov}\mathbf{Y}_t = \mathbb{E}(\mathbf{Y}_t - \mathbb{E}\mathbf{Y}_t)(\mathbf{Y}_t - \mathbb{E}\mathbf{Y}_t)' = \boldsymbol{\Sigma}_k$ for $\tau_{k-1}^\star + 1 \leqslant t \leqslant \tau_k^\star$.
- We consider the case of changes in the covariance matrix (the mean of the process is assumed constant)
- There exist a m-dimensional vector μ such that $\mathbb{E}\mathbf{Y}_t = \mu$ pour $t = 1, 2, \ldots, T$. Further, $\boldsymbol{\Sigma}_k \neq \boldsymbol{\Sigma}_{k+1}$ for $1 \leqslant k \leqslant K^\star - 1$

Global method: Contrast function III

- Change in the covariance matrix - constant mean (volatility models)
- Gaussian contrast

$$J(\boldsymbol{\tau}, \mathbf{Y}) = \frac{1}{T} \sum_{k=1}^{K} n_k \log |\widehat{\boldsymbol{\Sigma}}_{\tau_k}|,$$

  - $n_k = \tau_k - \tau_{k-1}$ is the length of the segment k
  - $\widehat{\boldsymbol{\Sigma}}_{\tau_k}$: $(m \times m)$ empirical covariance matrix evaluated on the segment k:

$$\widehat{\boldsymbol{\Sigma}}_{\tau_k} = \frac{1}{n_k} \sum_{t=\tau_{k-1}+1}^{\tau_k} (\mathbf{Y}_t - \bar{\mathbf{Y}})(\mathbf{Y}_t - \bar{\mathbf{Y}})', \quad \bar{\mathbf{Y}} = T^{-1} \sum_{t=1}^{T} \mathbf{Y}_t$$

Global method: Contrast function IV

- Univariate framework by Lavielle
- Similar rates of convergence
- Original adaptive method for determining the penalty parameter

Global method: Contrast function V

Asymptotic results for the minimum contrast estimator of $\boldsymbol{\tau}^\star$ are obtained in the following framework:

A1

For all $1 \leqslant i \leqslant m$ and $1 \leqslant t \leq T$, define $\eta_{t,i} = Y_{t,i} - \mathbb{E}Y_{t,i}$. There exists $C > 0$ and $1 \leqslant h < 2$ such that for any $u \geq 0$ and $s \geq 1$,

$$\mathbb{E}\left(\sum_{t=u+1}^{u+s} \eta_{t,i}\right)^2 \leqslant C(\theta)s^h.$$

(**A1** is verified with $h = 1$ for weakly dependent series, and $1 < h < 2$ for strongly dependent series.)

A2

There exists a sequence $0 < a_1 < a_2 < \ldots < a_{K^\star-1} < a_{K^\star} = 1$ such that for any $T \geqslant 1$ and for any $1 \leqslant k \leqslant K^\star - 1$, $\tau_k^\star = [Ta_k]$.

Global method: Contrast function VI

When the true number $K^\star$ of segments is known, we have the following result concerning the rate of convergence of the minimum contrast estimator of $\boldsymbol{\tau}^\star$:

Theorem

Assume that conditions **A1-A2** are satisfied. Let $\hat{\boldsymbol{\tau}}_T$ the times that minimize the empirical contrast. Then, the sequence $\{T\|\hat{\boldsymbol{\tau}}_T - \boldsymbol{\tau}^\star\|_\infty\}$ is uniformly tight in probability:

$$\lim_{T\to\infty} \lim_{\delta\to\infty} \mathrm{P}(\max_{1\leqslant k\leqslant K^\star-1} |\hat{\tau}_{T,k} - \tau_k^\star| > \delta) = 0$$

Remark

K is usually unknown, so that we have to estimate the dimension of the model.

Global method: Contrast function VII

Change-point times are estimated by minimizing the penalized contrast function

$$J(\boldsymbol{\tau}, \mathbf{y}) + \beta \mathrm{pen}(\boldsymbol{\tau}) = J(\boldsymbol{\tau}, \mathbf{y}) + \beta_T K$$

where

1. $\beta_T K$: penalty term that controls the level of resolution of the segmentation $\boldsymbol{\tau} = \{\tau_1, \tau_2, \ldots, \tau_{K-1}\}$.
2. If β is a function of T that goes to 0 at an appropriate rate as T goes to infinity, the following theorem states that the estimated number of segments converges in probability to the real number of segments $K^\star$

Global method: Contrast function VIII

Theorem

Let $\{\beta_T\}$ be a positive sequence of real numbers such that

$$\beta_T \underset{T \to \infty}{\longrightarrow} 0 \quad \text{and} \quad n^{2-h}\beta_T \underset{T \to \infty}{\longrightarrow} \infty.$$

Then, under **A1-A2**, the estimated number of segments $K(\hat{\boldsymbol{\tau}}_T)$, where $\hat{\boldsymbol{\tau}}_T$ is the minimum penalized contrast estimate of $\boldsymbol{\tau}^\star$ obtained by minimizing $J(\boldsymbol{\tau}, \mathbf{Y}) + \beta_T \mathrm{pen}(\boldsymbol{\tau})$, converges in probability to $K^\star$.

Penalty term

Standard choices for β (over-estimate the number of changes) :

- $\beta_T = \log(T)/T$ (Bayes Information Criteria)
- $\beta_T = 4\log(T)/T^{1-2d}$ for strongly dependent series,
- How to estimate the unknown d from real data ?
  - Spectral estimators over-estimate d and then artificially increase β.
  - Wavelet methods require large samples (issue of lowest octave selection)
- Adaptive method: the segmentation does not depend too much on β
- Consider the curve (K, J_K): we select the dimension K so that J_K ceases to decrease significantly

Adaptive choice for the penalty parameter I

$$\begin{aligned} J_K &= J(\hat{\boldsymbol{\tau}}_K, \mathbf{Y}), \\ p_K &= \mathrm{pen}(\boldsymbol{\tau}), \quad \forall \boldsymbol{\tau} \in \mathcal{T}_K \\ \hat{p}_K &= \mathrm{pen}(\hat{\boldsymbol{\tau}}_K). \end{aligned}$$

For any penalization parameter $\beta > 0$, the solution $\hat{\boldsymbol{\tau}}(\beta)$ minimizes the penalized contrast:

$$\begin{aligned} \hat{\boldsymbol{\tau}}(\beta) &= \arg\min_{\boldsymbol{\tau}} (J(\boldsymbol{\tau}, \mathbf{Y}) + \beta \mathrm{pen}(\boldsymbol{\tau})) \\ &= \hat{\boldsymbol{\tau}}_{\hat{K}(\beta)} \end{aligned}$$

where

$$\hat{K}(\beta) = \arg\min_{K \geqslant 1} \{J_K + \beta p_K\}.$$

Adaptive choice for the penalty parameter II

- The solution $\hat{K}(\beta)$ is a piecewise constant function of β.
- More precisely, if $\hat{K}(\beta) = K$,

$$J_K + \beta p_K \quad < \quad \min_{L \neq K}(J_L + \beta p_L).$$

- Thus, β satisfies

$$\max_{L>K} \frac{J_K - J_L}{p_L - p_K} < \beta < \min_{L<K} \frac{J_L - J_K}{p_K - p_L}.$$

- Then, there exists a sequence $\{K_1 = 1 < K_2 < \ldots\}$, and a sequence $\{\beta_0 = \infty > \beta_1 > \ldots\}$, with

$$\beta_i = \frac{J_{K_i} - J_{K_{i+1}}}{p_{K_{i+1}} - p_{K_i}} \ , \quad i \geqslant 1,$$

such that $\hat{K}(\beta) = K_i$, $\forall \beta \in [\beta_i, \beta_{i-1})$.

- Furthermore, the subset $\{(p_{K_i}, J_{K_i}), i \geqslant 1\}$ is the convex hull of the set $\{(p_K, J_K), K \geqslant 1\}$.

Adaptive choice for the penalty parameter III

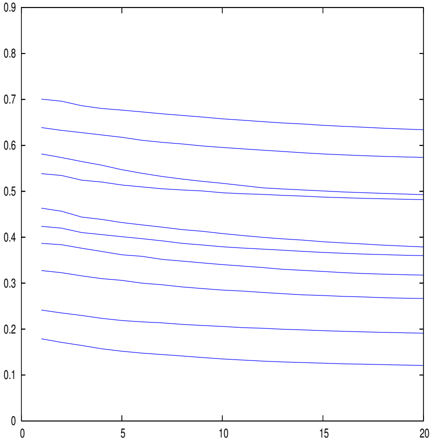
In summary, we propose the following procedure:

1. for $K = 1, 2, \ldots, K_{MAX}$, compute $\hat{\boldsymbol{\tau}}_K$, $J_K = J(\hat{\boldsymbol{\tau}}_K, \mathbf{Y})$ and $p_K = \mathrm{pen}(\hat{\boldsymbol{\tau}}_K)$,
2. compute the sequences $\{K_i\}$ and $\{\beta_i\}$, and the lengths $\{l_{K_i}\}$ of the intervals $[\beta_i, \beta_{i-1})$,
3. retain the greatest value(s) of K_i such that $l_{K_i} \gg l_{K_j}$, for $j > i$.

Adaptive choice for the penalty parameter IV

- Method difficult to automatize
- Consider another approach for selecting the dimension of the model
- Method that provides very good results and very easy to automate for practical applications
- Idea of the method: model the decrease of the sequence $\{J_K\}$ when there is no change in the series $\{\mathbf{Y}_t\}$ and look for which value of K this model adjusts the sequence of observed contrast
- Without changes in the variance, the joint distribution of $\{J_K\}$ is very difficult to model analytically
- However, Monte Carlo simulations shows that this sequence decreases as $c_1 K + c_2 K \log(K)$.

Adaptive choice for the penalty parameter V



- Ten sequences of contrast functions $\{J_K\}$ computed from 10 sequences of i.i.d. Gaussian random variables with correlation coefficient $\rho = 0.5$
- The fit with the function $c_1 K + c_2 K \log(K)$ is almost perfect ($r^2 > 0.999$).
  (the estimated coefficients $\hat{c}_1$ et $\hat{c}_2$ are different for each of these series).

Algorithm for the adaptive choice for the penalty parameter

Algorithm

For $i = 1, 2, \ldots,$

1. fit the model

$$J_K = c_1 K + c_2 K \log(K) + e_K,$$

to the sequence $\{J_K, K \geqslant K_i\}$, assuming that $\{e_K\}$ is a sequence of i.i.d. centered Gaussian random variables,

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2. evaluate the probability that $J_{K_i - 1}$ follows also this model, i.e., estimate the probability
$$\mathcal{P}_{K_i} = P\big(e_{K_i-1} \geqslant J_{K_i-1} - \hat{c}_1(K_i - 1) + \hat{c}_2(K_i - 1)\log(K_i - 1)\big),$$
under this estimated model.

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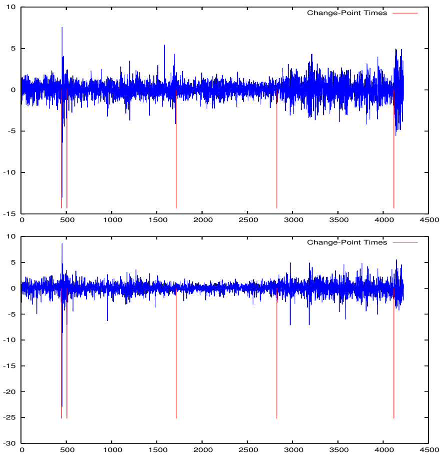
under this estimated model.
3. Then, the estimated number of segments will be the largest value of K_i such that the P–value $\mathcal{P}_{K_i}$ is smaller than a given threshold α.
(We set $\alpha = 10^{-7}$ and $K_{MAX} = 20$ in the numerical examples.)

Application to the bivariate series FT100 and S&P 500



- Adaptive detection of the number of change-points
- Above: FTSE 100
- Below: S&P 500

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- From simulated multivariate data, the BIC criteria strongly overestimates the number of change–points
- On real data (non Gaussian), the automatic method detects only the main changes (stock-market crashes, etc)
- However, this method is interactive as the user can choose a more "realistic" configuration by choosing a suitable P–value $\mathcal{P}_{K_i}$

Long-memory revisited I

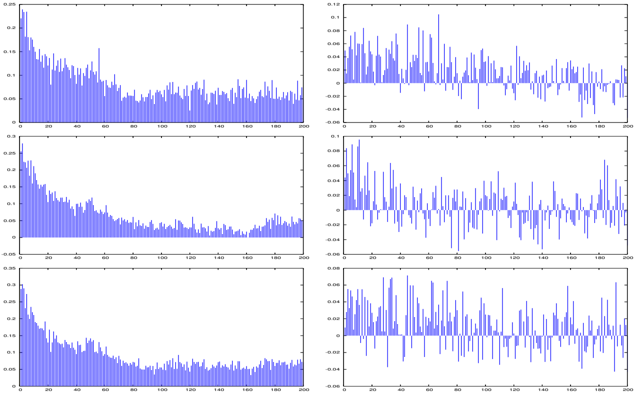

Figure: Left column: From top to bottom the sample autocorrelations on absolute returns on S&P 500 ($|r_S|$), absolute returns on FTSE 100 ($|r_F|$), and the sequence of their co–volatility $\sqrt{|r_S r_F|}$ for the whole sample.
Right Column: The sample ACF of these series for the time interval [508 : 1715]

Long-memory revisited II

- Long memory appears to be present in some time series (stock indices)
- But the intensity of strong dependence is lower than what is usually claimed
- This is consistent with what we get with the Increment Ratio Statistic (see the previous presentation by Donatas Surgailis)

Monte Carlo experiment I

Consider the constant conditional correlation bivariate GARCH, used for modeling multivariate time series:

$$\begin{pmatrix} Y_{1,t} \\ Y_{2,t} \end{pmatrix} = \boldsymbol{\Sigma}_t^{\frac{1}{2}} \begin{pmatrix} \varepsilon_{1,t}, \\ \varepsilon_{2,t}, \end{pmatrix}, \quad \begin{pmatrix} \varepsilon_{1,t}, \\ \varepsilon_{2,t}, \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right],$$

where the diagonal components of $\boldsymbol{\Sigma}_t$ are time varying and are univariate GARCH(1,1) processes:

$$\boldsymbol{\Sigma}_t = \begin{pmatrix} \sigma_{1,t}^2 & \rho\sigma_{1,t}\sigma_{2,t} \\ \rho\sigma_{1,t}\sigma_{2,t} & \sigma_{2,t}^2 \end{pmatrix}, \qquad \begin{array}{l} \sigma_{1,t}^2 = \omega_1 + \beta_1\sigma_{1,t-1}^2 + \alpha_1 Y_{1,t-1}^2 \\ \sigma_{2,t}^2 = \omega_2 + \beta_2\sigma_{2,t-1}^2 + \alpha_2 Y_{2,t-1}^2 \end{array}.$$

The coefficient of correlation ρ is constant, $\rho \in (-1, 1)$

Monte Carlo experiment II

- Locally stationary bivariate GARCH process

$$\begin{array}{rcl} \sigma_{1,t}^2 & = & \omega_1 + \beta_1 \sigma_{1,t-1}^2 + \alpha_1 Y_{1,t-1}^2 \\ \sigma_{2,t}^2 & = & \omega_2 + \beta_2 \sigma_{2,t-1}^2 + \alpha_2 Y_{2,t-1}^2 \end{array}, \quad \rho = 0.5, \quad t = 1, \dots, \tau_1,$$

$$\begin{array}{rcl} \sigma_{1,t}^2 & = & \bar{\omega}_1 + \bar{\beta}_1 \sigma_{1,t-1}^2 + \bar{\alpha}_1 Y_{1,t-1}^2 \\ \sigma_{2,t}^2 & = & \bar{\omega}_2 + \bar{\beta}_2 \sigma_{2,t-1}^2 + \bar{\alpha}_2 Y_{2,t-1}^2 \end{array}, \quad \rho = 0.3, \quad t = \tau_1 + 1, \dots, \tau_2,$$

$$\begin{array}{rcl} \sigma_{1,t}^2 & = & \tilde{\omega}_1 + \tilde{\beta}_1 \sigma_{1,t-1}^2 + \tilde{\alpha}_1 Y_{1,t-1}^2 \\ \sigma_{2,t}^2 & = & \tilde{\omega}_2 + \tilde{\beta}_2 \sigma_{2,t-1}^2 + \tilde{\alpha}_2 Y_{2,t-1}^2 \end{array}, \quad \rho = 0.7, \quad t = \tau_2 + 1, \dots, T.$$

- At time τ_1 all parameters of the process change, while at time τ_2, only ρ changes.
  $\omega_1 = 0.1$, $\beta_1 = 0.3$, $\alpha_1 = 0.2$, $\omega_2 = 0.15$, $\beta_2 = 0.2$, $\alpha_2 = 0.2$, $\tilde{\omega}_1 = \bar{\omega}_1 = 0.2$, $\tilde{\beta}_1 = \bar{\beta}_1 = 0.1$, $\tilde{\alpha}_1 = \bar{\alpha}_1 = 0.1$, $\tilde{\omega}_2 = \bar{\omega}_2 = 0.05$, $\tilde{\beta}_2 = \bar{\beta}_2 = 0.3$, $\tilde{\alpha}_2 = \bar{\alpha}_2 = 0.2$.

Monte Carlo experiment III

Table: Average number of detected change–points and their location using the Schwarz criteria, $T = 500, \tau_1 = 200, \tau_2 = 350$. Std errors between parentheses

DGP	Nb of change–points	$\hat{\tau}_1$	$\hat{\tau}_2$
No Changes	2.1626 (1.47)	—	—
Two Changes	3.8324 (1.55)	145.6920 (73.07)	243.7830 (100.99)

Table: Average number of detected change–points and their location using the adaptive method, $T = 500, \tau_1 = 200, \tau_2 = 350$. Std errors between parentheses

DGP	Nb of change–points	$\hat{\tau}_1$	$\hat{\tau}_2$
No changes	0.2962 (0.90)	—	—
Two changes	1.5650 (0.83)	217.1770 (64.31)	330.1390 (61.25)

Adaptive method detects with accuracy the number and location of changes

Current extensions

- In that case we considered multiple common change–points in multivariate time series
- We are now considering the case of non-common change points (work commissioned by EDF, French Electricity Company)

References

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